Discovering Zero and Negative Exponents	Directions: Complete the table with a partner. Show both ways. Try to follow patterns and use what you know about laws of exponents.		Name: **Key**
Evaluate the power.	**Take the previous answer and divide by 2.** (Decompose and find equivalent forms of 1)	**Show using laws of exponents.**	**What happens?**
$2^4 = 16$ $2 \cdot 2 \cdot 2 \cdot 2 = 16$	$\frac{2^4}{2^1} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2}$ $= 2 \cdot 2 \cdot 2$ $= 2^3$	$\frac{2^4}{2^1} = 2^{4-1}$ $= 2^3$	The exponent decreases by 1.
$2^3 = 8$ $2 \cdot 2 \cdot 2 = 8$	$\frac{2^3}{2^1} = \frac{2 \cdot 2 \cdot 2}{2}$ $= 2 \cdot 2$ $= 2^2$	$\frac{2^3}{2^1} = 2^{3-1}$ $= 2^2$	The exponent decreases by 1.
$2^2 = 4$ $2 \cdot 2 = 4$	$\frac{2^2}{2^1} = \frac{2 \cdot 2}{2}$ $= 2$ $= 2^1$	$\frac{2^2}{2^1} = 2^{2-1}$ $= 2^1$ $= 2$	The exponent decreases by 1.
$2^1 = 2$	$\frac{2^1}{2^1} = \frac{2}{2}$ $= 1$	$\frac{2^1}{2^1} = \frac{2}{2}$ $= 2^{1-1}$ $= 2^0$	The exponent decreases by 1 and we see that $2^0 = 1$.
$2^0 = 1$	$\frac{2^0}{2^1} = \frac{1}{2^1}$ $= \frac{1}{2}$	$\frac{2^0}{2^1} = 2^{0-1}$ $= 2^{-1}$	The exponent decreases by 1 and we see that $2^{-1} = \frac{1}{2}$.
$2^{-1} = \frac{1}{2}$	$\frac{2^{-1}}{2^1} = \frac{\frac{1}{2}}{2}$ $= \frac{1}{2} \cdot \frac{1}{2}$ $= \frac{1}{4}$	$\frac{2^{-1}}{2^1} = 2^{-1-1}$ $= 2^{-2}$	The exponent decreases by 1 and we see that $2^{-2} = \frac{1}{4}$.
$2^{-2} = \frac{1}{4}$	$\frac{2^{-2}}{2^1} = \frac{\frac{1}{4}}{2}$ $= \frac{1}{4} \cdot \frac{1}{2}$ $= \frac{1}{8}$	$\frac{2^{-2}}{2^1} = 2^{-2-1}$ $= 2^{-3}$	The exponent decreases by 1 and we see that $2^{-3} = \frac{1}{8}$.

You Try! Complete the table below using what you now know about a zero exponent or a negative exponent.

NOTE: (Have these pre-made on chart paper so that students may come up and fill in solutions as they are found.)

5^x	10^x	
5^4 = $\mathbf{625}$	10^4 = $\mathbf{10000}$	**Make an observation**: Describe some of the patterns you see going down the columns. • **We are dividing the base repeatedly by 5 or 10.** • **Powers with opposite exponents are reciprocals.** • **None of the answers are negative despite the negative exponents.**
$5^3 = \mathbf{125}$	$10^3 = \mathbf{1000}$	
$5^2 = \mathbf{25}$	$10^2 = \mathbf{100}$	
$5^1 = \mathbf{5}$	$10^1 = \mathbf{10}$	
$5^0 = \mathbf{1}$	$10^0 = \mathbf{1}$	
$5^{-1} = \mathbf{\frac{1}{5}}$	$10^{-1} = \mathbf{\frac{1}{10}}$	
$5^{-2} = \mathbf{\frac{1}{25}}$	$10^{-2} = \mathbf{\frac{1}{100}}$	
$5^{-3} = \mathbf{\frac{1}{125}}$	$10^{-3} = \mathbf{\frac{1}{1000}}$	

Negative Exponents

A) Compare the two powers: 2^1 and 2^{-1}. How would you describe the two answers? **(CHORAL RESPONSE)**
"They are reciprocals."

B) Compare the two powers: 2^2 and 2^{-2}. How would you describe the two answers? **(CHORAL RESPONSE)**
"They are reciprocals."

Make a prediction: Based on the last questions, if $2^{-1} = \frac{1}{2}$, and $2^{-2} = \frac{1}{4}$, and $2^3 = 8$ then $2^{-3} = \mathbf{\frac{1}{8}}$

Therefore, we have determined that a negative exponent is like a command. It tells us to:

Find the reciprocal of the base…before we evaluate the power.

Note: Negative exponents do not yield negative answers. (Unless, in some cases, when the base itself is negative.)

For every non-zero number a and integer, n, $a^{-n} = \frac{1}{a^n}$.

Directions: Simplify. **(Debrief having students post solutions on half transparencies or on chart paper)**

Example: $\mathbf{6^{-1}}$ $=\left(\frac{6}{1}\right)^{-1}$ $=\left(\frac{1}{6}\right)^{1}$ $=\frac{1}{6}$	You try! $\mathbf{11^{-1}}$ $=\left(\frac{11}{1}\right)^{-1}$ $=\left(\frac{1}{11}\right)^{1}$ $=\frac{1}{11}$	You try! $\boldsymbol{x^{-1}}$ $=\left(\frac{x}{1}\right)^{-1}$ $=\left(\frac{1}{x}\right)^{1}$ $=\frac{1}{x}$
Example: $\mathbf{(-2)^{-1}}$ $=\left(-\frac{2}{1}\right)^{-1}$ $=\left(-\frac{1}{2}\right)^{1}$ $=-\frac{1}{2}$	You try! $\mathbf{(-5)^{-1}}$ $=\left(-\frac{5}{1}\right)^{-1}$ $=\left(-\frac{1}{5}\right)^{1}$ $=-\frac{1}{5}$	You Try! $\boldsymbol{(-x)^{-1}}$ $=\left(-\frac{x}{1}\right)^{-1}$ $=\left(-\frac{1}{x}\right)^{1}$ $=-\frac{1}{x}$
Example: $\boldsymbol{5t^{-2}}$ $=5\cdot t^{-2}$ $=5\cdot\left(\frac{t}{1}\right)^{-2}$ $=5\cdot\left(\frac{1}{t}\right)^{2}$ $=5\cdot\left(\frac{1}{t}\right)\cdot\left(\frac{1}{t}\right)$ $=5\cdot\left(\frac{1\cdot 1}{t\cdot t}\right)$ $=5\cdot\frac{1}{t^2}$ $=\frac{5}{t^2}$	You Try! $\boldsymbol{(5t)^{-2}}$ $=\left(\frac{5t}{1}\right)^{-2}$ $=\left(\frac{1}{5t}\right)^{2}$ $=\left(\frac{1}{5t}\right)\cdot\left(\frac{1}{5t}\right)$ $=\frac{1\cdot 1}{5t\cdot 5t}$ $=\frac{1}{5\cdot 5\cdot t\cdot t}$ $=\frac{1}{25t^2}$	You try! $\boldsymbol{x^{-3}y}$ $=x^{-3}\cdot y$ $=\left(\frac{x}{1}\right)^{-3}\cdot y$ $=\left(\frac{1}{x}\right)^{3}\cdot y$ $=\left(\frac{1}{x}\right)\cdot\left(\frac{1}{x}\right)\cdot\left(\frac{1}{x}\right)\cdot y$ $=\frac{1\cdot 1\cdot 1}{x\cdot x\cdot x}\cdot y$ $=\frac{1}{x^3}\cdot y$ $=\frac{y}{x^3}$

Zero Exponent

C) What did you notice when dividing $\frac{2^1}{2^1}$? (Hint, you should have found two answers, 2^0 and 1.)

We found that $\frac{2^1}{2^1} = 1$ and $\frac{2^1}{2^1} = 2^0$ therefore, $2^0 = 1$.

D) **Make a prediction:** What is the answer to a problem like $\frac{3^1}{3^1}$ or $\frac{x^1}{x^1}$?

The answer would be 1 or $3^0 = 1$ and $x^0 = 1$.

E) Therefore, we have determined that an exponent of zero always gives an answer of: **(CHORAL RESPONSE) One!**

For every non-zero number a, $a^0 = 1$.

Raising any Base to a Zero Power

Simplify $(xy^2z^3)^0$

We may apply two rules of exponents.

$$(xy^2z^3)^0 = (x)^0(y^2)^0(z^3)^0$$ ← Power of a Product Rule

$$= x^0y^{2\cdot 0}z^{3\cdot 0}$$ ← Power of a Power Rule

We may now apply the Zero Exponent Rule.

$$= x^0y^0z^0$$
$$= 1$$

Directions: Simplify using exponent rules. **(Debrief using THINK-PAIR-SHARE)**

Example: $\frac{2^4}{2^4}$ $= 2^{4-4}$ $= 2^0$ 1	You try! $\frac{x^3}{x^3}$ $= x^{3-3}$ $= x^0$ $= 1$	You try! $\frac{11^6}{11^6}$ $= 11^{6-6}$ $= 11^0$ $= 1$
Example: z^0 $= 1$	You try! $\left(\frac{x}{5}\right)^0$ $= \frac{x^0}{5^0}$ $= \frac{1}{1}$ $= 1$	You try! $(abc^2)^0$ $= (a)^0(b)^0(c^2)^0$ $= a^0b^0c^{2\cdot 0}$ $= a^0b^0c^0$ $= 1 \cdot 1 \cdot 1$

Raising a Quotient to a Negative Power

Simplify $\left(\frac{2}{3}\right)^{-2}$

We should first find the reciprocal of $\frac{2}{3}$.

After this, we evaluate the power.

Method #1

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{2}{3}\right)^{-1\cdot 2}$$
$$= (\left(\frac{2}{3}\right)^{-1})^2$$
$$= \left(\frac{3}{2}\right)^2$$
$$= \left(\frac{3}{2}\right)\left(\frac{3}{2}\right)$$
$$= \frac{9}{4}$$

Notice: When we have found the reciprocal, the exponent becomes positive.

Why?
We have completed the action of taking the reciprocal.

In Method #2 we evaluated the power first and took the reciprocal at the end of the problem.

Method #2

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{2}{3}\right)^{2\cdot -1}$$
$$= (\left(\frac{2}{3}\right)^2)^{-1}$$
$$= \left(\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\right)^{-1}$$
$$= \left(\frac{4}{9}\right)^{-1}$$
$$= \frac{9}{4}$$

Example 3: Evaluate using laws of exponents and negative and zero exponents.
(Remember, a negative exponent indicates that we should find the reciprocal and THEN evaluate the power.)

NOTE: Ask students to show multiple methods on these problems if there is time. Display to class as well.

Example: $\left(\frac{5}{3}\right)^{-3}$	You try! $\left(\frac{y}{x^2}\right)^{-5}$	You try! $\left(\frac{4x}{12x^2y}\right)^{-2}$
$=\left(\frac{5}{3}\right)^{-1\cdot 3}$	$=\left(\frac{y}{x^2}\right)^{-1\cdot 5}$	$=\left(\frac{4x}{12x^2y}\right)^{-1\cdot 2}$
$=\left(\left(\frac{5}{3}\right)^{-1}\right)^3$	$=\left(\left(\frac{y}{x^2}\right)^{-1}\right)^5$	$=\left(\left(\frac{4x}{12x^2y}\right)^{-1}\right)^2$
$=\left(\frac{3}{5}\right)^3$	$=\left(\frac{x^2}{y}\right)^5$	$=\left(\frac{12x^2y}{4x}\right)^2$
$=\left(\frac{3}{5}\right)\cdot\left(\frac{3}{5}\right)\cdot\left(\frac{3}{5}\right)$	$=\left(\frac{x^2}{y}\right)\cdot\left(\frac{x^2}{y}\right)\cdot\left(\frac{x^2}{y}\right)\cdot\left(\frac{x^2}{y}\right)\cdot\left(\frac{x^2}{y}\right)$	$=\left(\frac{12x^2y}{4x}\right)\cdot\left(\frac{12x^2y}{4x}\right)$
$=\frac{3\cdot 3\cdot 3}{5\cdot 5\cdot 5}$	$=\frac{x^2\cdot x^2\cdot x^2\cdot x^2\cdot x^2}{y\cdot y\cdot y\cdot y\cdot y}$	$=\frac{12x^2y\cdot 12x^2y}{4x\cdot 4x}$
$=\frac{3^3}{5^3}$	$=\frac{(x^2)^5}{y^5}$	$=\frac{12\cdot 12\cdot x^2\cdot x^2\cdot y\cdot y}{4\cdot 4\cdot x\cdot x}$
$=\frac{27}{125}$	$=\frac{x^{2\cdot 5}}{y^5}$	$=\frac{144x^4y^2}{16x^2}$
	$=\frac{x^{10}}{y^5}$	

Example: $\frac{1}{3^{-2}}$	You try! $\frac{1}{2^{-3}}$	You Try! $\frac{x}{3^{-3}}$
$=\frac{\frac{1}{3^{-2}}}{1}$	$=\frac{\frac{1}{2^{-3}}}{1}$	$=\frac{\frac{x}{3^{-3}}}{1}$
$=\frac{1}{\left(\frac{3}{1}\right)^{-1\cdot 2}}$	$=\frac{1}{\left(\frac{2}{1}\right)^{-1\cdot 3}}$	$=\frac{x}{\left(\frac{3}{1}\right)^{-1\cdot 3}}$
$=\frac{1}{\left(\left(\frac{3}{1}\right)^{-1}\right)^{2}}$	$=\frac{1}{\left(\left(\frac{2}{1}\right)^{-1}\right)^{3}}$	$=\frac{x}{\left(\left(\frac{3}{1}\right)^{-1}\right)^{3}}$
$=\frac{1}{\left(\frac{1}{3}\right)^{2}}$	$=\frac{1}{\left(\frac{1}{2}\right)^{3}}$	$=\frac{x}{\left(\frac{1}{3}\right)^{3}}$
$=\frac{1}{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}$	$=\frac{1}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$	$=\frac{x}{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}$
$=\frac{1}{\frac{1}{9}}$	$=\frac{1}{\frac{1}{8}}$	$=\frac{x}{\frac{1}{27}}$
$=1\cdot\frac{9}{1}$	$=1\cdot\frac{8}{1}$	$=x\cdot\frac{27}{1}$
$=9$	$=8$	$=27x$